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**ON THE METHODS OF INVESTIGATING THE TRAJECTORIES OF THE
MOTION OF OBJECTS OF LABOR IN THE PHASE SPACE OF STATES**

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Abstract

A model of an economic-production system with a flow-based method for organizing production is considered. The space of states of the subject of labor is introduced. The state of a particular subject of labor is determined by a point in the state space. The state of the economic-production system is determined through the aggregation of the states of a large number of the subjects of labor that are in the process of production. The paper assumes that if the state of each individual subject of labor is known, then the state of the parameters of the economic-production system is known. Using the variational and differential principles, Lagrange's function of the economic-production system is recorded. The equation for the normative technological trajectory of the movement of a subject of labor along a technological route is defined in the state space. When the subject of labor moves along a technological trajectory from one technological operation to another technological operation, technological resources are transferred to the subject of the labor as a result action of technological equipment. The Lagrange function of the economic-production system with a flow-based method of organizing production is recorded. The Lagrange function is constructed using the variational and differential principle. Methodological differences are shown when using the variational and differential approach to construct the Lagrange function of economic-production systems. It is shown that the Lagrange function, taking into account production and technical and socio-economic constraints, determines the subjective function of the economic-production system. The integrals of the movement of subjects of labor in the state space are defined, which can be used to model economic-production systems with a flow-based method of organizing production. It is shown that the integrals of motion are conservation laws that characterize the process of processing subjects of labor along the technological route.

Keywords: subjective function; variational principle; differential principle; production system; technological operation; basic product; acceptable deviations; technological trajectory; production system.

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**О МЕТОДАХ ИССЛЕДОВАНИЯ ТРАЕКТОРИЙ ДВИЖЕНИЯ
ПРЕДМЕТОВ ТРУДА В ФАЗОВОМ ПРОСТРАНСТВЕ СОСТОЯНИЙ**

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Аннотация

Рассмотрена модель экономико-производственной системы с поточным методом организации производства. Введено пространство состояний предмета труда. Состояние отдельного предмета труда определяется точкой в пространстве состояний. Состояние экономико-производственной системы определяется через агрегирование состояний большого количества предметов труда, находящихся в процессе производства. В работе предполагается, что если известно состояние каждого отдельно взятого предмета труда, то известно состояние параметров экономико-производственной системы. С использованием вариационного и дифференциального принципов записана функция Лагранжа экономико-

производственной системы. Определено в пространстве состояний уравнение для нормативной технологической траектории движения предмета труда по технологическому маршруту. При движении предмета труда по технологической траектории от одной технологической операции к другой технологической операции на предмет труда переносятся технологические ресурсы в результате воздействия технологического оборудования. Записана функция Лагранжа экономико-производственной системы с поточным методом организации производства. Для построения функции Лагранжа использован вариационный и дифференциальный принцип. Показаны методологические отличия при использовании вариационного и дифференциального подхода для построения функции Лагранжа экономико-производственных систем. Показано, что функция Лагранжа с учетом производственно-технических и социально-экономических ограничений определяет целевую функцию экономико-производственной системы. Определены интегралы движения предметов труда в пространстве состояний, которые могут быть использованы для моделирования экономико-производственных систем с поточным методом организации производства. Показано, что интегралы движения являются законами сохранения, характеризующие процесс обработки предметов труда вдоль технологического маршрута.

Ключевые слова: целевая функция; вариационный принцип; дифференциальный принцип; производственная система; технологическая операция; базовый продукт; допустимые отклонения; технологический шум; производственная система.

1. INTRODUCTION

The description of the functioning of modern production can be represented in the form of a process in which the production system passes from one of its states to another. The state of the system can be defined as the state of the total number N of basic products of the production system [1, p.178]. Under the basic product (or conventional product [2, p.183]) is understood the element of the production system, to which the transfer of the value of living labor, raw materials, materials and tools during its movement along the operating chain of technological maps occurs. Along this movement, a task-oriented transformation of the starting and raw materials (interoperable procurement) into a finished product through the directed impact of socially useful labor takes place. The state of the j -base product can be described by microscopic quantities (engineering and manufacturing arguments) (S_j, μ_j) [3], where

S_j (\$) and $\mu_j = \lim_{\Delta t \rightarrow 0} \frac{\Delta S_j}{\Delta t}$ (\$/hour), respectively, the sum of total costs and costs per unit of time transferred by the production system to the j -base product, $0 < j \leq N$. The considered production system [4, 5] will be characterized by the function $J_{II}(t, S_i, \mu_i)$ [6]. Through the S_0 and μ_0 variables the technology of manufacturing process of the base product in the phase technologic space (S, μ) of the observed engineering and manufacturing arguments is defined. Variables S_0 and μ_0 determine the standard technologic technological trajectory for the production process [6, 7, 8]:

$$S_0 = S_0(t), \quad \mu_0 = \frac{dS_0(t)}{dt} \quad (1)$$

in phase space (S, μ) . The nominal technologic technological trajectory is determinate. Determinacy of the nominal technological trajectory follows the uniqueness of the given technology of manufacturing of the base product¹ [9-10]. Each manufacturing process has nominal technological arguments of production of the base product and acceptable deviation from the nominal technological arguments. The manufacturing process, performed with exceeding of the values of the maximum deviation, leads to a

¹ ГОСТ 3.1109.82. Термины и определения основных понятий. – М.: Госстандарт России, 2003. – 15 с.
GOST 3.1109.82. Terms and definitions of basic concepts. – М.: Gosstandart of Russia, 2003. – 15 p. (in Russia)

violation of the manufacturing process, that is, to the emerging of a reject workpiece² [9-10]. Let us assume that the manufacturing process at each m technological process be defined by k technology factors with their design intent $\langle Z_{m,k} \rangle - \Delta Z_{2m,k} \leq Z_{m,k} \leq \langle Z_{m,k} \rangle - \Delta Z_{1m,k}$, $m=1..N_m$, $\kappa=1..N_k$, where N_m и N_k denote, thereafter, the number of process stages and the number of technology factors that is allowed by the process stage. Each technology factor $Z_{m,k}$ is a random process with expectation function (face value) $\langle Z_{m,k} \rangle$ and thereafter upper and lower acceptable deviation (operational margin) $\Delta Z_{1m,k}$ and $\Delta Z_{2m,k}$ from the statutory value. The upper and lower operational margin $\Delta Z_{1m,k}$ and $\Delta Z_{2m,k}$ determine the upper and lower divergences of the parameter of the process stage from the given standard technologic trajectory allowed by the technology of manufacturing of the base product. When manufacturing the base product with technological parameters within the limits of the upper $\Delta Z_{1m,k}$ and lower $\Delta Z_{2m,k}$ deviations from the statutory values, allowed by the manufactory process, it is recognized that the base product is manufactured in accordance with the defined technology. Then, using the means of the theory of random processes³, the values of technological parameters could be received $\mu_{0m} - \sigma_{\mu_{0m}} \leq \mu_m \leq \mu_{0m} + \sigma_{\mu_{0m}}$, $S_{0m} - \sigma_{S_{0m}} \leq S_m \leq S_{0m} + \sigma_{S_{0m}}$ with m for process stage

$$\mu_{0m} = \mu_{0m}(Z_{m,k}, \Delta Z_{1m,k}, \Delta Z_{2m,k}), \quad \sigma_{\mu_{0m}} = \sigma_{\mu_{0m}}(Z_{m,k}, \Delta Z_{1m,k}, \Delta Z_{2m,k}), \quad \kappa=1..N_k \quad (2)$$

2. MAIN PART

With a large number of process stages $N_m \gg 1$, it is convenient to go from a discrete description of values μ_{0m} , $\sigma_{\mu_{0m}}$ of the manufactory process to continuous description of values $\mu_0(t)$, $\sigma_\mu(t)$ on the multitude of integrable and differentiable functions [3]. Note that the values $S_0(t)$, $\sigma_S(t)$ can be obtained by the method of integrating technological parameters $\mu_0(t)$, $\sigma_\mu(t)$. Each j base product in the process of processing treatment passes from its reference state (initial procurement) to the final state (end product) in accordance with the given manufacturing process of the base product and forms a technologic technological trajectory in the phase technologic space (S, μ) . This technologic technological trajectory is the implementation of the manufactory process for the j base product. The technological process is implemented in the neighborhoods of the known manufacturing process of the base product, which is limited by the zone of admissible technologic trajectories.

The manufacturing process is determined by the plurality of states of the basic products. If it is known everything about the state of each base product at any particular time, then it is reasonable to assume that everything is also known about the state of the production system. The change in time of the properties of each base product of the production system can be represented as the movement of the base product in the phase space of the observed production and technologic parameters (S, μ) , and the law of motion can be obtained by using the methods of the variational calculus. Let us assume that at the instants of time $t=t_1$ and $t=t_2$ the system is in a state $J_{II} = (t_1, S_{j1}, \mu_{j1})$, $S_{j1} = S_j(t_1)$ and in state

² ГОСТ 3.1121.84. Общие требования к комплектности и оформлению на типовые и групповые технологические процессы (операции). – М.: Госстандарт России, 2005. – 48 с.

GOST 3.1121.84. General requirements for completeness and registration for typical and group technological processes (operations). – Moscow: Gosstandart of Russia, 2005. – 48 p. (in Russia)

³ ГОСТ 14.004-83. Технологическая подготовка производства. Термины и определения основных понятий. – М.: Госстандарт России, 2007. – 4 с.

GOST 14.004-83. Technological preparation of production. Terms and definitions of basic concepts. – Moscow: Gosstandart of Russia, 2007. – 4 p. (in Russia)

$J_{\Pi} = (t_2, S_{j_2}, \mu_{j_2})$, $S_{j_2} = S_j(t_2)$ thereof. Then, between these states, the implementation of the technological process must be carried out in such a way that the subjective functional

$$I = \int_{t_1}^{t_2} J_{\Pi} dt \quad (3)$$

has a minimum relatively to possible deviations from the given technology.

Such an approach, which in the construction of the laws of motion of mechanical systems is called the principle of least action, is used in the construction of economic theories of production systems [11, 12]. The variation of the subjective functional (3) allows us to determine the equations of motion of each separately taken base product in the phase space of the observed engineering and manufacturing parameters (S, μ) [13-16]. Integrating the equations of motion for each base product, we obtain information about the state parameters of each base product, and hence information about the state of the production system as a whole [1, 17, 18]. Since everything is known about the state of each base product, then everything is known about the state of the production system [7, 8]. In most cases, the subjective functional (3) contains parameters that describe random factors of the implementation of the technological process [1, 3, 8, 15]. In this case, with the variation of the subjective functional (3), the equations of motion of the base product, which include random functions of implementation of the technological process are obtained [19]. Thus, as a result of integrating the equations of motion, a technological trajectory for the j base product, which is the implementation of the technological process and is determined by randomly generated factors during the movement of the base product along the process chain is obtained.

3. RESEARCH METHODS

3.1 THE VARIATIONAL PRINCIPLE OF CONSTRUCTING OF THE SUBJECTIVE FUNCTION OF THE PRODUCTION SYSTEM

Let the standard technological trajectory (1) of the manufacturing of the base product in the phase space (S, μ) be determined by the growth function of costs during the movement of the base product along the process chain. growth function of costs is built on the basis of process charts of the technological process, which determine the sequence of operations of the action of production of the base product and necessary production resources for performing manufacturing process (raw materials, labor, electricity, etc.) [3, 19]. growth function of costs – describes the process of accumulation of costs in accordance with the chosen technology of manufacturing of the base product [20]. We require that the subjective functional (3) has an extremal value on the set of possible trajectory $S_j(t, \alpha)$ for the manufacturing process with the given production technology of the base product.

The subjective functional (3), enumerated along a specific technological trajectory, is a function of the parameter α :

$$I(\alpha) = \int_{t_1}^{t_2} J_{\Pi}(t, S_j(t, \alpha), \mu_j(t, \alpha)) dt . \quad (4)$$

We calculate the fluctuation of the functional (3):

$$\delta I = \int_{t_1}^{t_2} \delta J_{\Pi} dt = \int_{t_1}^{t_2} \sum_{j=1}^N \left(\frac{\partial J_{\Pi}}{\partial S_j} \delta S_j + \frac{\partial J_{\Pi}}{\partial \mu_j} \delta \mu_j \right) dt = \int_{t_1}^{t_2} \sum_{j=1}^N \left(\frac{\partial J_{\Pi}}{\partial S_j} - \frac{d}{dt} \frac{\partial J_{\Pi}}{\partial \mu_j} \right) \delta S_j dt . \quad (5)$$

The integral is transformed by integration by parts:

$$\delta \mu_j = \delta \frac{d}{dt} S_j(t, \alpha) = \frac{\partial}{\partial \alpha} \frac{d}{dt} S_j(t, \alpha) \delta \alpha = \frac{d}{dt} \frac{\partial}{\partial \alpha} S_j(t, \alpha) \delta \alpha = \frac{d}{dt} \delta S_j \quad (6)$$

Technological trajectory $S_j(t, \alpha)$ for an individual j base product of the production system has a common beginning $(t_1, S_j(t_1))$ and a common ending $(t_2, S_j(t_2))$. Therefore, for $t = t_1$ and with $t = t_2$ the variations δS_j are equal zero and the integrated part becomes zero.

Since the implementation of the technological process must be carried out in such a way that the subjective functional (3) for the production process has a minimum, then the variation of the subjective functional (5) is equal zero:

$$\int_{t_1}^{t_2} \sum_{j=1}^N \left(\frac{\partial J_{\Pi}}{\partial S_j} - \frac{d}{dt} \frac{\partial J_{\Pi}}{\partial \mu_j} \right) \delta S_j dt = 0, \quad (7)$$

which determines the equations of Euler for each individual base product

$$\frac{\partial J_{\Pi}}{\partial S_j} - \frac{d}{dt} \frac{\partial J_{\Pi}}{\partial \mu_j} = 0. \quad (8)$$

For production systems is known both the technology of production of the base product, and the criteria characterizing the deviation of technological parameters of the base product as it moves along the technological chain. The last is enough to construct the subjective function $J_{\Pi}(t, S_j, \mu_j)$ in an explicit form and write the equations of Euler (8) for each individual base product, which determine the states of the base product when it moves from one technological operation to another.

The equations of Euler for each individual base product (8) can also be obtained by differential equations of motion of the basic products along the process chain. However, there is one fundamental difference between differential equations and variational principles: the differential equations express some functional connection that connecting the position of the base products along the production chain of the production system, the transfer rate of cost and the acceleration of the cost transfer to the base product at the instant of time t . The variational principle also characterizes the normative technological process as a whole. It formulates the stationary property of the subjective functional for a given technological process. The variational principle has a more visible and compact form and is often used as a basis for constructing new methods for describing systems [7, 13, 21, 22].

3.2 DIFFERENTIAL PRINCIPLE OF BUILDING OF THE SUBJECTIVE FUNCTION OF THE PRODUCTION SYSTEM.

Euler differential equation for the basic products of the production system (8) is necessary and sufficient conditions for the variation be equal zero (6). Let the movement of the base product along the technological chain can be described by the equations [18, 19]:

$$\frac{d\mu_j(t)}{dt} = f(S_j), \quad \frac{dS_j(t)}{dt} = \mu_j, \quad (9)$$

where through the $f(S_j)$ indicated rate of change of the rate of transfer of costs to the j base product. In mechanics, this intensity is called a summarized force. We obtain the Euler equations (8) from the general equation of the dynamics of the production process:

$$\sum_{j=1}^N \left(f(S_j) - \frac{d^2 S_j(t)}{dt^2} \right) \delta S_j = 0. \quad (10)$$

Using the permutability of the operations of variation and differentiation with respect to time, we obtain

$$\sum_{j=1}^N \frac{d^2 S_j(t)}{dt^2} \delta S_j = \frac{d}{dt} \sum_{j=1}^N \frac{dS_j(t)}{dt} \delta S_j - \sum_{j=1}^N \frac{dS_j(t)}{dt} \frac{d}{dt} \delta S_j = \frac{d}{dt} \sum_{j=1}^N \frac{dS_j(t)}{dt} \delta S_j - \sum_{j=1}^N \frac{dS_j(t)}{dt} \frac{d\delta S_j}{dt} \quad (11)$$

where

$$\frac{d}{dt} \delta S_j = \delta \mu_j = \delta \frac{dS_j}{dt}.$$

We introduce the integral characteristics $F(S_j)$ of the rate of change of costs

$$F(S_j) = \int_0^{S_j} f(S) dS \quad (12)$$

Then it follows that

$$\sum_{j=1}^N f(S_j) \delta S_j = -\delta \sum_{j=1}^N F(S_j) \quad (13)$$

We write the equation of the dynamics of the production process of the base product (10) in the form

$$\delta \sum_{j=1}^N \frac{\mu_j^2(t)}{2} - \frac{d}{dt} \sum_{j=1}^N S_j(t) \delta S_j - \delta \sum_{j=1}^N F(S_j) = 0 \quad (14)$$

We integrate both sides of this equation in the range from $t = t_1$ to $t = t_2$

$$\int_{t_1}^{t_2} \left(\delta \sum_{j=1}^N \frac{\mu_j^2(t)}{2} - \delta \sum_{j=1}^N F(S_j) \right) dt - \left(\sum_{j=1}^N \frac{dS_j(t)}{dt} \delta S_j \right)_{t=t_1}^{t=t_2} = 0. \quad (15)$$

Since the variation δS_j for $t = t_1$ and $t = t_2$ equals zero, hence:

$$\left(\sum_{j=1}^N \frac{dS_j(t)}{dt} \delta S_j \right)_{t=t_1}^{t=t_2} = 0, \quad \int_{t_1}^{t_2} \left(\delta \sum_{j=1}^N \frac{\mu_j^2(t)}{2} - \delta \sum_{j=1}^N F(S_j) \right) dt = 0, \quad (16)$$

and congruence (16) can be transformed to the form

$$\delta \int_{t_1}^{t_2} \left(\sum_{j=1}^N \frac{\mu_j^2(t)}{2} - \sum_{j=1}^N F(S_j) \right) dt = 0, \quad (17)$$

where

$$J_{\Pi} = \sum_{j=1}^N \frac{\mu_j^2(t)}{2} - \sum_{j=1}^N F(S_j) \quad (18)$$

he subjective function of the production system.

Thus, the general equation of dynamics (10) led us to the variational principle

$$\delta \int_{t_1}^{t_2} J_{\Pi} dt = 0$$

4. ANALYSIS OF THE RESULTS

When the basic products of the production system move along the technological chain in the phase space (S, μ) , there are functions $\varphi(S_j, \mu_j)$ of economic magnitudes S_j, μ_j that preserve constant values, depending only on the initial conditions, during the motion of the system. Such functions will be called the first integrals of the motion of the production system.

If the subjective function of the production system does not depend explicitly on time, then the total derivative of it can be written in the form:

$$\frac{dJ_{\Pi}}{dt} = \sum_{j=1}^{N_1} \left[\frac{\partial J_{\Pi}}{\partial S_j} \frac{dS_j}{dt} \right] + \sum_{j=1}^{N_1} \left[\frac{\partial J_{\Pi}}{\partial \mu_j} \frac{d\mu_j}{dt} \right] \quad (19)$$

By virtue of the equation of Euler (9), we replace the derivatives $\frac{\partial J_{\Pi}}{\partial S}$ by their values $\frac{d}{dt} \left(\frac{\partial J_{\Pi}}{\partial \mu_j} \right)$, we obtain

$$\frac{d}{dt} \sum_{j=1}^{N_2} \left[\left(\frac{\partial J_{\Pi}}{\partial \mu_j} \right) \frac{dS_j}{dt} - J_{\Pi} \right] = 0. \quad (20)$$

From which the intensity

$$\sum_{j=1}^{N_2} \left[\left(\frac{\partial J_{\Pi}}{\partial \mu_j} \right) \frac{dS_j}{dt} - J_{\Pi} \right] = \text{const} \quad (21)$$

is constant at the movement of base products along a technological chain. The systems, having an integral of this type, are called conservative.

The next integral of the motion of the production system arises from the likeness of the phase space. As consequence of the likeness of the phase space with respect to the coordinate S, we require that the subjective function $J_{\Pi}(t, S_i, \mu_i)$ of the determined system remains unchanged when the system is transferred to a segment δS , as a whole. The change in the subjective function due to a small displacement along the phase coordinate S:

$$\delta J_{\Pi}(t, S_j, \mu_j) = \sum_{j=1}^{N_1} \left[\frac{\partial J_{\Pi}}{\partial S_j} \delta S \right] = \delta S \sum_{j=1}^{N_1} \frac{\partial J_{\Pi}}{\partial S_j}. \quad (22)$$

By virtue of arbitrariness δS it follows that $\delta J_{\Pi} = 0$ and it means that: the sum of all the technological influences on the basic products of the production system equals zero.

$$\sum_{j=1}^{N_1} \frac{\partial J_{\Pi}}{\partial S_j} = 0. \quad (23)$$

Then, by the equations of Euler (8), we obtain

$$\sum_{j=1}^{N_1} \frac{d}{dt} \left(\frac{\partial J_{\Pi}}{\partial \mu_j} \right) = \frac{d}{dt} \sum_{j=1}^{N_1} \left(\frac{\partial J_{\Pi}}{\partial \mu_j} \right) = 0, \quad (24)$$

from which

$$P_S = \sum_{j=1}^{N_1} \left(\frac{\partial J_{\Pi}}{\partial \mu_j} \right) = \text{const}. \quad (25)$$

An analogous integral of the motion of the production system arises from the invariance of the phase space with respect to the phase velocity

$$\sum_{j=1}^{N_1} \frac{\partial J_{\Pi}}{\partial \mu_j} = 0 \quad (26)$$

and is a necessary condition for the extremum of the microparameters of the production system.

From the equations of Euler (9), the properties of the subjective function are seen. If the production system consists of two non-interacting parts (production sites, departments, locations [23, 24]), then the congruence is correct:

$$J_{\Pi} = J_{\Pi_1} + J_{\Pi_2} \quad (27)$$

The multiplying of the subjective function of the production system by an arbitrary constant does not change the equations of motion of the base products, but leads only to the appropriate choice of the system of units of measure for building the model of the production system.

The subjective function of the production system is determined accurate to the total derivative of any function of the coordinates $S_j(t)$ in time $t: \theta(t, S_j)$. The latest is relevant to the fact that the variation from the function $\theta_{II}(t, S_j)$ is the null equation:

$$\delta\theta_{II}(t, S_j)dt = \int_{t_1}^{t_2} \left[\frac{\partial\theta_{II}}{\partial S_j} \delta S_j \right] dt = \delta S_j \int_{t_1}^{t_2} \frac{\partial\theta_{II}}{\partial S_j} dt = 0 \quad (28)$$

SUMMARY

Using the variational and differential principles, the subjective function for the basic products of the production system is recorded. The terms of the subjective function that characterize the technological field of the equipment and the intrinsic properties of the base product are determined. The first integrals of the movement of the base product along the technological chain are recorded. The properties of the subjective function for the basic products of the production system are shown. The resulting equations can be widely used in the design of highly efficient production line management systems, both with the use of production and technological description, and with the use of PDE-models of production systems [25, 26]. This approach is a competitive alternative to the use of Clearing functions [16, 26] for modeling the changes in the state of flow parameters of production lines of modern industrial enterprises.

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